time the negative electron must be free from molecules of the gas. When X/P increases, M becomes smaller still, but as the velocities have not yet been determined accurately for higher values of X/P, further investigation on these lines must be postponed.

In order that the equations of diffusion should hold in these cases, it is necessary that the velocity of the ion in the direction of the electromotive force should be small compared with the velocity of agitation. It is easy to show that this condition holds. Since the velocity of agitation of a molecule of CO_2 at 0° centigrade is 4×10^4 centimetres per second, the velocity of particles of an average mass 1/43 of that of the molecules of CO_2 would be $2{\cdot}8 \times 10^5$, and while under the electric force their velocity of agitation would be $2{\cdot}8 \times 10^5 \times \sqrt{1{\cdot}85} = 3{\cdot}8 \times 10^5$, which is large compared with the velocity, 1845 centimetres per second, in the direction of the electric force.

On the Energy and Distribution of Scattered Röntgen Radiation.

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(Communicated by Prof. Sir J. J. Thomson, F.R.S. Received December 13, 1910,—Read January 26, 1911.)

Introduction.

It has been shown by Prof. Sir J. J. Thomson that it is possible, on certain assumptions, to calculate the number of electrons in an atom of any element from considerations of the scattering undergone by a rapidly moving electrified particle in the substance. In a paper recently read before this Society* I was able, by means of experiments there described, to show that the scattering of homogeneous β-rays by thin sheets of different substances did follow very closely the laws predicted by the theory, and to calculate the number of electrons contained in the atoms of some half dozen different elements. It was found that, in every case, the number of electrons per atom was equal to about three times the atomic weight.

Although the agreement between theory and experiment was very close, it seemed desirable to collect as much independent evidence as possible upon such a point. Some years ago Prof. Sir J. J. Thomson† showed that in the phenomenon of the scattering of a beam of Röntgen rays we have another possible method of attacking the problem.

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* 'Roy. Soc. Proc.,' 1910, A, vol. 84, p. 226.

† 'Conduction through Gases,' 1906, p. 321.

Assuming that the scattered Röntgen radiation is emitted by the electrons in the atoms traversed by the primary rays executing forced vibrations in the period of the incident rays, it can easily be shown that the energy radiated per electron is equal to

$$\frac{8\pi}{3}\frac{e^4}{m^2}\,.\,\mathrm{E},$$

where E is the energy in the primary pulse, and e and m the charge and mass of the electron. The total energy radiated by a given mass of substance is therefore equal to

$$\frac{8\pi}{3}\mathrm{N}\frac{e^4}{m^2}\,.\,\mathrm{E},$$

where N is the total number of electrons in the given mass. This equation gives us an independent method of arriving at the number of electrons in the atom.

So far as I am aware, no determinations of the energy of scattered Röntgen radiation have been made since the early experiments of Barkla* on the secondary radiation from gases. He compared the ionisation in two similar electroscopes, one placed in the path of the primary rays, the other in a direction at right angles to it. He thus compared the energy of the secondary rays in a given fixed direction with that of the incident beam. To deduce the total secondary radiation he assumed a law of distribution deduced from the simple theory described above. As no experimental proof of this law of distribution had been given, and in view of the new light which has been thrown upon the whole subject since these early experiments were performed, it was thought desirable that a fresh determination of the energy of the scattered Röntgen radiation should be made.

It was not found feasible to measure immediately the whole of the rays scattered by a given radiator. The experiment therefore resolved itself into two parts:—

(1) The comparison of the intensity of the scattered radiation from a given radiator in a fixed plane (namely, that at right angles to the primary beam).

(2) The determination of the distribution of the scattered radiation around the radiator.

The following sections of this paper describe the methods employed and the results obtained.

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* Barkla, 'Phil. Mag.,' 1904, [6], vol. 7, p. 543.

On the Energy of the Röntgen Radiation Scattered in a Plane at Right Angles to the Primary Beam.

The direct mechanical measurement of the energy of Röntgen radiation is practically impossible. In order to compare the intensities of two given streams of Röntgen radiation, we must rely on their power either to affect a photographic plate or to ionise a gas. Of the two methods, the latter is the more definite, and the less open to objection. In fact, if the two beams have the same penetrating power, and pass through the ionisation chambers without striking any solid object, it seems reasonably certain that the amount of ionisation produced per unit length of path in the chamber will be simply proportional to the energy of beams.

If, however, the rays fall upon the electrodes and walls of the ionisation chamber, the bulk of the ionisation produced is due to the emission of corpuscular radiation from the substances struck. The amount of this will depend upon the shape and material of the chamber, and the way in which the rays strike it. Results with two different chambers will therefore not in general be strictly comparable. This objection, however, does not apply if the *same* ionisation chamber is used to measure successively different quantities of radiation, providing all pass through the chamber in the same way.

As it was desirable, on account of possible variations in the X-ray tube, to make simultaneous measurements of the primary and scattered radiations, and to eliminate as many sources of uncertainty as possible, it was decided to arrange matters so that neither the primary nor the secondary rays should impinge on any solid object in the vicinity of the ionisation chambers. The effect to be measured was thus greatly reduced, but it was thought that this was more than compensated for by the increased certainty that the measurements of the two beams were really comparable.

The methods employed will be readily understood from fig. 1, which represents a section of the apparatus. The X-ray tube F supplying the rays is completely enclosed in a thick lead box, to eliminate any fear of stray radiations; the leads from the coil were introduced through thick porcelain tubes. A narrow pencil of rays, limited by a series of circular apertures A, A, passes upwards through the lead tubes T, T, so that no radiation can emerge into the secondary ionisation chamber except at the point R, where a gap 2·5 cm. in length is left for the radiator. On its way, the beam passes through the small primary ionisation chamber P, fitted with two parallel aluminium electrodes e, e', well out of the path of the rays. The measuring electrode e is provided with a guard ring in order to ensure a uniform field across the chamber.

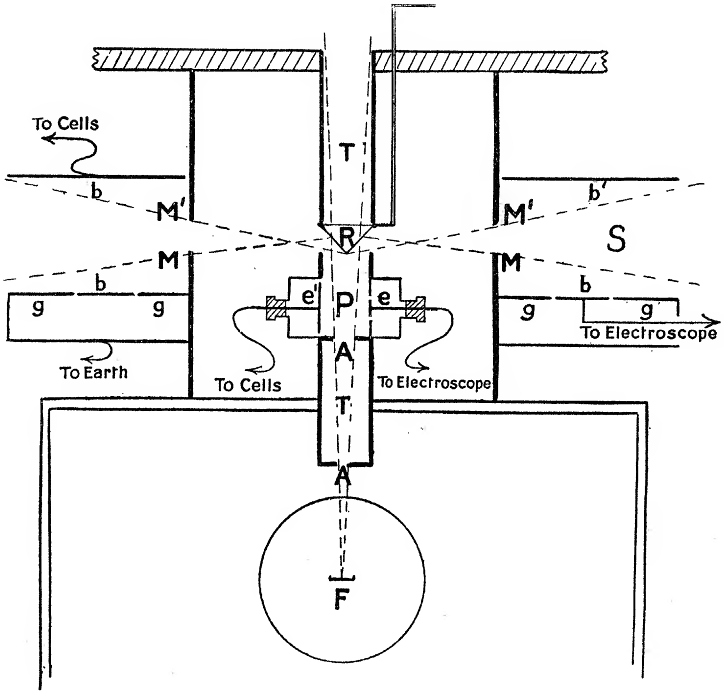

Fig. 1.

The rest of the apparatus is symmetrical about the line of the primary beam, which it completely surrounds like a belt. MM and M′M′ are two lead cylinders, 24 cm. in diameter and 2·5 cm. apart, and serve to limit the secondary rays from R to a fairly narrow sheet. The secondary ionisation chamber S is fitted with two parallel plate electrodes *b*, *b′*; the measuring electrode *b* being surrounded by guard rings *g*, *g*, as in the case of the primary chamber P. Wires passing through earthed tubes, not shown in the diagram, connect the electrodes *b* and *e* to two Wilson electroscopes; *b′* and *e′* are charged to saturation potentials from a cabinet of accumulators.

The radiator R took the form of a hollow cone of aluminium foil, with a vertical angle of 90°. The primary rays thus met it everywhere at an angle of 45°, and the secondary rays emerged at the same angle. The scattered radiation from the air was not sufficient to measure with any accuracy.

As the chamber S was large, there was a very appreciable spontaneous ionisation current across it. To eliminate this, a cylindrical ionisation chamber, charged to the opposite potential, was connected to the same electroscope. The effective volume of this compensator could be varied by means of an earthed tube sliding over the inner electrode. By varying the length of the electrode exposed, the two currents could be made to neutralise each other.

In making the measurements, readings were taken with the rays passing, but with no radiator at R, and then with the radiator in position. The difference gave the effect due to the radiator alone. The effect in the absence of the radiator was always very small.

By means of a key, not shown in the figure, the electrode b could be directly connected to the electrode e of the primary chamber. In this way the capacities of the two systems could be directly compared.

To compare the relative amounts of energy in the primary and secondary beams, we may proceed as follows:—

Let E_P be the energy per unit volume in the primary beam,
α_P its cross-section, and
d_P the effective length of path in the ionisation chamber; that is to say, the length of path in that portion of the gas of which the ionisation is measured.

The volume of gas ionised is thus, $\alpha_P d_P$, and the rate of ionisation is therefore $kE_P\alpha_P d_P$, where k is a constant depending on the nature of the gas and the hardness of the rays. Similarly, for the secondary beam, we have the rate of ionisation equal to $kE_S\alpha_S d_S$, where k is the same factor as before, since the secondary rays are of the same hardness as the primary.

If q is the number of ions of one sign produced in a time t, the charge conveyed to the electrode in that time is equal to qe, where e is the charge on an electron. If the capacity of the electrode is c, and the potential acquired in the time t is equal to v, we have $qe = cv$, thus the rate of production of ions $= q/t = cv/et$.

Equating these expressions, we have

$$kE_S\alpha_S d_S = \frac{c_S v_S}{et}, \qquad kE_P\alpha_P d_P = \frac{c_P v_P}{et}$$

so that

$$\frac{E_S\alpha_S}{E_P\alpha_P} = \frac{v_S}{v_P} \cdot \frac{c_S}{c_P} \cdot \frac{d_P}{d_S}.$$

Since for a limited beam the energy crossing any cross-section of the beam in a given time is constant, $E\alpha$ is constant for a given beam of radiation.

The following are the experimental values:—

$$d_P = 1{\cdot}0 \text{ cm.}, \qquad d_S = 5{\cdot}0 \text{ cm.}, \qquad c_S/c_P = 1{\cdot}33.$$

Rate of charging up of the two systems in volts per minute :—

v_S.	v_P.	v_S/v_P.
0·074	12·0	0·0062
0·070	11·2	0·0063
0·068	10·5	0·0065
0·068	11·0	0·0062
0·069	11·2	0·0062
0·078	12·2	0·0064
	Mean value.........	0·0063

Thus $$\frac{E_S\alpha_S}{E_P\alpha_P} = \frac{v_S}{v_P} \cdot \frac{c_S}{c_P} \cdot \frac{d_P}{d_S} = 0·0063 \times 1·33 \times \tfrac{1}{5} = 0·00168.$$

The thickness of the aluminium radiator measured along the path of the primary rays was 0·109 mm. The angle of emergence of the secondary beam was 6° 24′ on each side of a plane through the radiator perpendicular to the primary beam.

The value given above has still to be corrected for the absorption of the primary and secondary rays in the radiator itself. Although the conical radiator was quite symmetrical about the primary beam, the equations obtained for the absorption in the radiator were not capable of explicit evaluation. The radiator was purposely made as thin as possible, in order to make the correction as small as possible. The following method was adopted for its determination: Consider the cross-section of the cone made by any plane perpendicular to its axis, and therefore to the primary beam. This cross-section will take the form of a ring, bounded by two concentric circles, whose distance apart is equal to $t\sqrt{2}$, where t is the thickness of the foil of which the cone is made. Consider any point P in this ring whose distance from the outer circle is y. Then the distance traversed by the primary rays in the cone in reaching P is also y, since the cone is inclined to both primary and secondary rays at an angle of 45°. The intensity of the primary rays at P is therefore $I_0e^{-\lambda y}$, where λ is the coefficient of absorption of the rays. For the rays used in these experiments, λ was equal to 11·0 cm.$^{-1}$.

Consider now the secondary rays from P in the plane of cross-section. When t is very small compared to the radius of either bounding circle, the ordinate through P at right angles to the radius becomes infinitely great compared to t. In the limit, therefore, the absorption is the same as if the radiator consisted of two parallel plates perpendicular to the radius through P. In practice the absorption must always be less than this limiting value.

If y is the depth of the point P below the surface of one of the plates, the path of a ray inclined at an angle θ to the normal is $y \sec \theta$.

If the initial intensity of radiation from P between the directions θ and $\theta+d\theta$ is $\mathrm{I}\,d\theta$, the emergent intensity is $\mathrm{I}\,d\theta\, e^{-\lambda y \sec\theta}$. The total emergent radiation is therefore $\mathrm{I}\int_0^{\frac{1}{2}\pi} e^{-\lambda y \sec\theta}\, d\theta$ for the rays emerging on the side nearest y. For those which pass through both plates the path is $(2t\sqrt{2}-y)\sec\theta$, and the emergent radiation is therefore $\mathrm{I}\int_0^{\frac{1}{2}\pi} e^{-\lambda(2t\sqrt{2}-y)\sec\theta}\, d\theta$. I have not been able to evaluate these integrals explicitly. They lend themselves, however, quite readily to evaluation by graphical methods. By actually plotting the graphs of $e^{-\lambda y \sec\theta}$ against θ for different values of y from $y = 0$ to $y = 0{\cdot}0109$ cm., and measuring the areas of the curves so obtained, a series of values were found for the absorption of the secondary rays, starting from different depths y in the plate. It has been shown that the absorption of the primary rays reaching a point P at a depth y in the radiator is $e^{-\lambda y}$.

Multiplying each of the values for the absorption of the secondary rays by the corresponding values of $e^{-\lambda y}$, we obtain, by a final graphical integration, the whole absorption of the rays in the material. It was found to be given by the equation

$$\mathrm{I}/\mathrm{I}_0 = 0{\cdot}78.$$

This represents the maximum absorption when the radius of the cross-section is large compared with the thickness of the foil. Turning now to the apex of the cone, where the cross-section is a circle of radius $t\sqrt{2}$, we can, by pursuing a similar course of operations, find the absorption for the cross-section of this shape. In this way it was found that the absorption for this case was given by

$$\mathrm{I}/\mathrm{I}_0 = 0{\cdot}84.$$

This represents the minimum absorption. The actual value must lie between these two. The values are not very far apart. We shall not commit any serious error if we take the true absorption in the radiator as being given by

$$\mathrm{I}/\mathrm{I}_0 = 0{\cdot}81.$$

I is the radiation as measured by the secondary chamber. To obtain the true value I_0, in the absence of any absorption in the radiator, we must multiply our result by $1/0{\cdot}81$, that is by $1{\cdot}24$. Making this correction, we get finally

$$\mathrm{E}_{\mathrm{S}}\alpha_{\mathrm{S}}/\mathrm{E}_{\mathrm{P}}\alpha_{\mathrm{P}} = 0{\cdot}00208.$$

Considering the beams at the point where their cross-section is unity, we have

$$\mathrm{E}_{\mathrm{S}}/\mathrm{E}_{\mathrm{P}} = 0{\cdot}00208.$$

On the Distribution of the Scattered Radiation round the Radiator.

In order to measure the distribution of the rays round the radiator it was necessary to design an ionisation chamber which could rapidly and accurately be brought to any desired position with respect to the primary beam and the radiator. At the same time, as a Wilson electroscope is very sensitive to slight changes of level, and other variations in external conditions, it was necessary, if accurate measurements were to be made, that the electroscope itself should remain fixed on a firm basis. The apparatus finally designed for the experiment is shown in fig. 2.

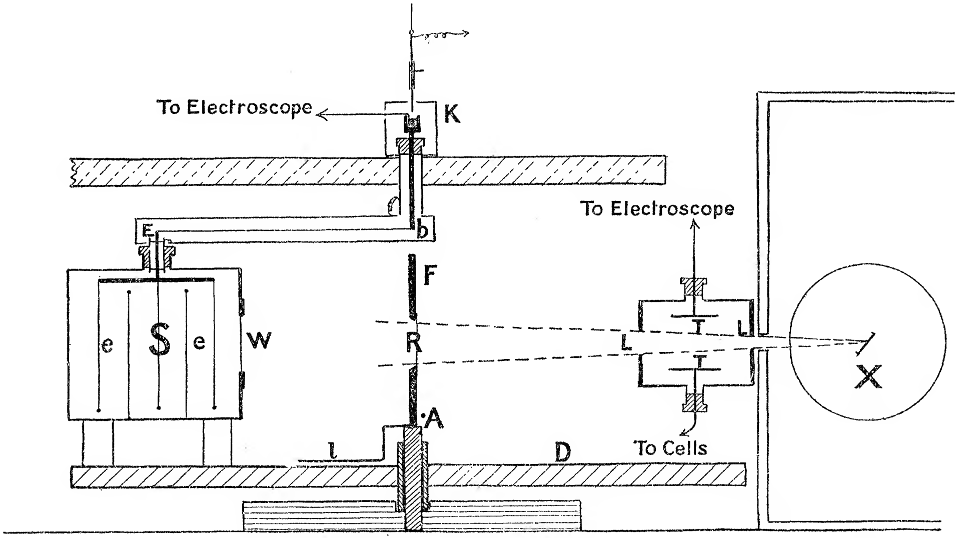

FIG. 2.

The ionisation chamber S used to measure the emergent scattered radiation is mounted upon the circumference of a large mahogany wheel D, which can be rotated about a fixed vertical axle A. The ionisation chamber S is cylindrical in shape, the cylinder being placed with its axis passing normally through the axis of rotation of the wheel. A window W, closed with thin aluminium foil, serves to admit a cone of the secondary rays of fixed angle (15°) into the ionisation chamber. A lead plate F, pierced with a suitable aperture to allow of the passage of the primary beam, is fixed to A, and serves to support the radiator R. In this way, by simply turning the wheel D, the secondary chamber S can be made to take up any position with respect to the radiator. The position of S can be read off on a circular scale fixed to D, and read by a pointer *l* attached to the fixed axle A.

As the amount of scattered radiation is very small, the ionisation chamber S was made fairly large, and in order to obtain saturation with a reasonably small voltage a system of plane parallel electrodes of aluminium leaf mounted on wire rings was introduced, as shown in the figure, instead of the more usual, but unsatisfactory, central wire electrode.

In order to make connection between the moving electrode E and the fixed electroscope, the key K was mounted so that the brass rod *b* came directly above the axis of rotation of the wheel D. The distance from E to *b* thus remains constant when the wheel is rotated, and the two can be joined together by a thin copper wire, shielded in an earthed metal tube in the usual way.

The focus tube X is enclosed as before in a lead box. A system of lead stops L, L, limits the emergent rays so that the cross-section of the beam at R is 3 cm. in diameter. This was verified by placing a fluorescent screen at R, when it was found that the beam passed through the aperture and was quite well defined. The primary beam passes between two parallel electrodes T, T, the ionisation between which can be measured in the usual way and serves to standardise the primary beam.

In making the measurements readings were made, as before, with and without the radiator R in position. The difference gave the true effect due to the radiator alone. Owing to the scattering of the rays in the air there was always a small but appreciable effect even when the radiator R was not present; but this was eliminated by the method of measurement described.

The radiator took the form of a sheet of the substance under experiment, the thickness being kept as small as possible, in order that the absorption of the rays in the radiator should be small. The absorption of the rays used by the substance was, however, always measured, and a correction calculated for every position of the ionisation chamber.

The following table contains a typical set of observations for an aluminium radiator. The first column of figures gives the angle θ made by the axis of S with the direction of the primary beam. R.H. and L.H. signify that the chamber is to the right or left hand, looking along the direction of the primary beam: "returned" denotes that the radiation is measured on the side of the radiator facing the incident rays; "transmitted" that it is measured on the further side. The second and third columns give the ionisation with and without the radiator, in arbitrary units, corrected for variations in the intensity of the primary beam. The fourth column is the difference of the second and third and gives the effect due to the radiator alone. The fifth column gives the correcting factor for the absorption of primary and secondary rays in the radiator. This has been calculated as follows:—Consider a small volume of unit area and thickness dx situated at a distance x from the front face of the radiator, which is struck normally by the primary beam. If I_0 is the initial intensity of the primary beam, the intensity on reaching the small volume is equal to $I_0e^{-\lambda x}$, where λ is the coefficient of absorption of the rays. Suppose now that the fraction $k\,.\,dx$ of this energy is scattered in a cone of small angle about the angle θ with the normal. The path of these rays before emerging from the face of the radiator for the returned radiation is $x \sec\theta$. and for the forward radiation $(a-x)\sec\theta$, where a is the thickness of the radiator. The energy of radiation emerging from a small volume at depth x is thus $kI_0e^{-\lambda x}\,e^{-\lambda x\sec\theta}\,dx$ in a return direction, and $k'I_0e^{-\lambda x}\,e^{-\lambda(a-x)\sec\theta}\,dx$ in a forward direction:—

	θ.	Ionisation.		Ionisation due to radiator.	Correction factor for absorption.	I_θ.	$I_\theta \sin\theta$.
		With radiator.	Without radiator.				
	°						
R.H. "returned" …	35	3·38	1·08	2·30	1·42	3·26	1·87
	50	2·95	0·90	2·05	1·50	3·06	2·34
	70	2·26	0·88	1·38	1·80	2·48	2·30
R.H. "transmitted"	25	11·10	3·63	7·47	1·40	10·50	4·45
	40	5·20	1·82	3·38	1·46	4·93	3·17
	60	3·54	1·54	2·00	1·67	3·34	2·90
L.H. "transmitted"	30	8·95	2·00	6·95	1·42	9·88	4·94
	50	4·10	1·03	3·07	1·55	4·76	3·65
	70	2·38	0·88	1·50	1·92	2·88	2·70
L.H. "returned" …	35	3·25	0·90	2·35	1·42	3·34	1·91
	50	2·20	0·85	1·85	1·50	2·78	2·14
	70	2·70	0·90	1·30	1·80	2·36	2·22

The whole intensity in the direction θ is thus given by

$$I_0k\int_0^a e^{-\lambda x(1+\sec\theta)}\,dx \quad \text{and} \quad I_0k'\int_0^a e^{-\lambda[x+(a-x)\sec\theta]}\,dx.$$

The intensity in the same directions in the absence of any absorption is simply kI_0a and $k'I_0a$ respectively. Evaluating the integrals, we find if I_θ, I'_θ represent the corrected and the measured values of the intensity in the direction θ,

$$\frac{I_\theta}{I'_\theta} = \frac{a\lambda(1+\sec\theta)}{1-e^{-\lambda a(1+\sec\theta)}} \quad \text{for the returned radiation,}$$

and

$$\frac{I_\theta}{I'_\theta} = \frac{a\lambda(\sec\theta-1)}{e^{-\lambda a}-e^{-\lambda a\sec\theta}} \quad \text{for the transmitted radiation.}$$

For the rays used λ was 11·0 cm.$^{-1}$.

The values of these expressions for the different values of θ are the correction factors given in the table in column 5. The sixth column gives the values of I_θ, the intensity of the scattered radiation in the direction θ; that is to say, the energy of the scattered radiation per unit area at an angle θ with the primary beam.

According to the simple theory of scattering outlined above, the distribution of the scattered radiation should be symmetrical both about the direction of the primary beam, and about a plane perpendicular to this direction. The intensity should be a minimum in this plane, and rise to a maximum on both sides of the radiator in the direction of the incident beam. The ratio of the maximum to the minimum intensities, assuming that the electrons are displaced in the plane of the wave-front, should be 2 to 1.

The actual distribution obtained differed considerably from this. Fig. 3 shows graphically the actual distribution around a small radiator R, the primary rays falling upon R in the direction of the arrow. Radii are drawn from R, showing the angle of inclination of the scattered rays with the normal, and lengths are marked off along them proportional to the intensity of radiation at that inclination. In this way we get a picture of the distribution of the rays about R. The same results are also shown as an ordinary graph in fig. 4.

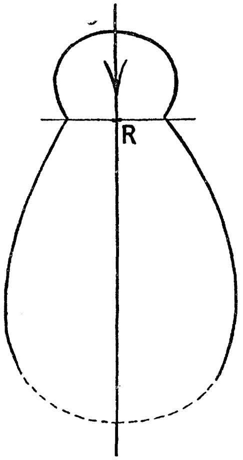

Fig. 3.

It will be seen from fig. 3 that the scattered radiation is symmetrical about the incident beam,* but not about the plane perpendicular to it, through the radiator.

It will be seen at once from both figures that while the intensity of the scattered radiations is a maximum both forward and backward along the line of the primary beam, and falls to a minimum in the plane perpendicular to this direction, the distribution about this plane is by no means symmetrical. At any given inclination to the primary beam the intensity

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* In these experiments the bulb was placed so that the path of the cathode rays in the bulb was perpendicular to the plane of rotation of the ionisation chamber. If the path of the cathode rays is in the plane of rotation, there is always a slight preponderance of the scattered radiation on that side of the primary beam to which the cathode rays are directed. This effect, which has been pointed out by several observers, is due to the partial polarisation of the primary rays. It was in order to eliminate any error due to this effect that the secondary chamber in the energy experiments already described was made to surround completely the primary beam.

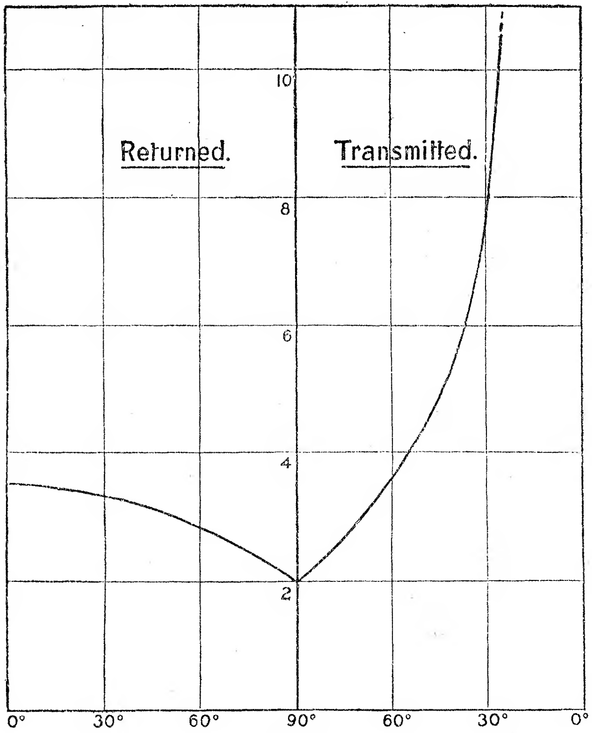

FIG. 4.

in the forward direction is always greater than that in the reverse direction, and this inequality becomes greater as the direction approaches more and more nearly to that of the primary beam. The distribution of the returned radiation agrees fairly well with that given by the simple theory. It was impossible to measure the returned radiation actually in the line of the primary beam. By interpolation, however, it was found that the intensity in this direction was about 3·64. The intensity at right angles to this was found to be 2·0; the ratio being 1·8 : 1. It seems, therefore, that while the maximum in the forward direction is greater than is required by the theory, the maximum in the reverse direction is somewhat less.

The form of curve suggested that the preponderance in the forward direction might be due to some other form of radiation superposed upon the true scattering. A considerable number of experiments have been made with a view to testing this point, but I have so far failed to obtain any evidence that such is the case. The distribution of the radiation does not seem to depend upon the thickness of the radiator, though it does to some extent upon the material, the eccentricity being rather less for paper than for aluminium. There was no appreciable difference in the hardness of the rays returned, and those transmitted, the relative intensities in the two directions being unaltered when sufficient aluminium was placed in front of W to cut down the radiation entering the ionisation chamber by more than one-half. The effects cannot be due to a penetrating "characteristic" radiation from the aluminium, as measurements on characteristic secondary radiation have shown that it is distributed equally at all angles round the radiator.* At present, therefore, we must regard the whole of the radiation as being purely scattered.

To obtain the whole energy of scattered radiation we may proceed as follows. Let R_θ be the intensity of the radiation at any angle θ with the primary beam, so that the energy included in a hollow cone of angle θ and increment $d\theta$ round the primary beam is equal to $R_\theta . d\theta$. The area swept out by this hollow cone on a sphere of radius r passing through the window of the ionisation chamber is $2\pi r \sin\theta . r d\theta = 2\pi r^2 \sin\theta . d\theta$. Now, if α is the area of the window, supposed to be sufficiently small to be included in this belt, the intensity of the rays I_θ entering the ionisation chamber is equal to

$$R_\theta \, d\theta \frac{\alpha}{2\pi r^2 \sin\theta \, d\theta}.$$

But α is constant, being the area of the window w, and r is constant. We can write therefore

$$R_\theta = \text{constant} \times I_\theta \sin\theta.$$

The values of $I_\theta \sin\theta$ are given in the last column of the previous table. The whole energy of the scattered radiation is equal to

$$\frac{1}{2\pi}\int_0^{2\pi} R_\theta . d\theta, \quad \text{that is to} \quad c' \int_0^{2\pi} I_\theta \sin\theta \, d\theta.$$

Similarly, the energy measured in the first part of the experiment is that included between angles of ± 6° 24′ with the plane perpendicular to the primary beam, *i.e.*,

$$c' \int_{\frac{1}{2}\pi - \phi}^{\frac{1}{2}\pi + \phi} I_\theta \sin\theta \, d\theta, \quad \text{where} \quad \phi = 6^\circ\ 24'.$$

We can integrate these quantities by a graphical method. A curve is plotted between $I_\theta \sin\theta$ and θ. The area of the whole curve gives the value of $\int_0^{2\pi} I_\theta \sin\theta \, d\theta$, that is, of the whole scattered radiation. The area included between the ordinates $\frac{1}{2}\pi \pm \phi$ gives in a similar way the energy between the angles $\frac{1}{2}\pi \pm \phi$. Drawing the curve we find that the whole area is 2292 in arbitrary units; the area between ordinates $\frac{1}{2}\pi \pm \phi$ is 138 on the same scale.

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* J. A. Crowther, 'Camb. Phil. Soc. Proc.,' November, 1910.

The energy measured in the first experiment is thus 138/2292, or 0·060 of the whole scattered radiation.

On the Energy of Scattered Radiation and the Number of Electrons in the Atom.

We are now in a position to form an estimate of the fraction of the energy of the primary Röntgen radiation scattered per unit mass of the radiator. We have found that 0·00208 of the energy of the primary beam is radiated between the angles ± 6° 24′ with the plane perpendicular to the primary beam by an aluminium radiator 0·109 mm. or 0·0109 cm. thick. We have also seen that this energy represents 0·060 of the whole scattered radiation. The total energy of scattered radiation must therefore be 0·00208/0·060 or 0·0347 of the energy in the primary beam.

The density of aluminium is 2·7; the mass per unit area of the radiator is therefore 0·0294 grammes per square centimetre. For a radiator of unit mass per square centimetre we should therefore have—

$$\frac{\text{Energy in scattered radiation}}{\text{Energy in primary radiation}} = \frac{0·0347}{0·0294} = \mathbf{1·18}.$$

It must be remembered that as we have corrected our results for the absorption of the primary rays this number represents the rate at which the primary radiation is being scattered. Representing the scattering by the equation

$$dS = kI \, . \, dw.$$

where S and I are the intensities of the scattered and primary rays, and w is the mass per unit area, the number 1·18 represents the factor k, which we may term the mass coefficient of scattering.

We have seen that the elementary theory of scattering does not entirely represent the actual distribution of the scattered radiation round the radiator. It seems probable, however, that the theory is sufficiently near the truth to give us a fairly reliable estimate of the energy scattered. Assuming that this is the case, we have

$$\frac{8\pi}{3} \, . \, N \frac{e^4}{m^2} = 1·18 \text{ per grm. per sq. cm.,}$$

where N is the total number of radiating electrons. Taking e as $1·55 \times 10^{-20}$ e.m.u. (Rutherford and Geiger) and e/m as $1·73 \times 10^7$ e.m.u. (Bucherer) we have

$$N = \frac{3}{8\pi} \cdot \frac{1·18}{(1·55 \times 10^{-20})^2 (1·73 \times 10^7)^2} = 1·95 \times 10^{24} \text{ per grm.}$$

Now the mass of a hydrogen atom is $1{\cdot}61 \times 10^{-24}$ (Rutherford and Geiger); the mass of an atom of aluminium is therefore

$$27 \times 1{\cdot}61 \times 10^{-24} = 4{\cdot}35 \times 10^{-23}$$

or the number of atoms of aluminium per gramme is

$$(4{\cdot}35 \times 10^{-23})^{-1} = 2{\cdot}30 \times 10^{22}.$$

The number of electrons N in this mass of aluminium as given by the scattering is $1{\cdot}95 \times 10^{24}$. The number of electrons per atom of aluminium is therefore found to be $\frac{1{\cdot}95 \times 10^{24}}{2{\cdot}30 \times 10^{22}}$ or **85.** The number of electrons per atom of aluminium obtained from the measurements on the scattering of homogeneous β-rays is 83.* The agreement between the two results arrived at from entirely different points of view is very satisfactory. It seems therefore that the estimate of the number of electrons in the atom as being three times the atomic weight is probably not far from the truth.

Summary.

The distribution of the scattered Röntgen radiation round a radiator has been measured. The intensity of the scattered radiation reaches a maximum on both sides of the radiator in the line of the primary beam, and falls to a minimum at right angles to this direction. At any given inclination to the primary beam there is a preponderance of radiation in the forward direction, the ratio increasing as the direction of the primary beam is approached.

The energy of the radiation scattered in a given direction has been compared with that in the primary beam. From the result so obtained, and the known distribution of the radiation, the total radiation scattered by an aluminium radiator has been determined.

It has been shown that the value obtained leads to the same result for the number of electrons in the atom as that previously calculated by the author from his experiments on the scattering of homogeneous β-rays.

In conclusion I wish once more to express my thanks to Prof. Sir J. J. Thomson for his inspiration and advice during the course of these experiments.

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* J. A. Crowther, 'Roy. Soc. Proc.,' 1910, A, vol. 84, p. 239.